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Numerical solution of mixed convective laminar boundary layer flow around a vertical slender body with suction or blowing

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Abstract: In this paper, the numerical solution of mixed convective laminar boundary layer flow around a vertical slender body with suction or blowing has been investigated. Firstly, the governing boundary layer partial differential equations have been made dimensionless and then simplified by using Boussinesq approximation. Secondly, similarity transformations are introduced on the basis of detailed analysis in order to transform the simplified coupled partial differential equations into a set of ordinary differential equations. The transformed complete similarity equations are solved numerically by using computer software. Finally, the flow phenomenon has been characterized with the help of obtained flow controlling parameters such as suction parameter, buoyancy parameter, Prandtl number, body-radius parameter and other driving parameters. Finally the effects of involved parameters on the velocity and temperature distributions are presented graphically. It is found that a small suction or blowing can play a significant role on the patterns of flow and temperature fields.

Keywords: Similarity Solution, Mixed Convection, Vertical Slender Body, Suction or Blowing

1. Introduction

The convective heat and mass transfer process takes place due to the buoyancy effects owing to the differences of temperature and concentration, respectively. In studying the fluid flow around a heated body it is usual to neglect the buoyancy effect when a forcing velocity exists. Under some circumstances, such an assumption is not reasonable, the buoyancy forces considerably modifying the boundary layer characteristics. In dealing with the transport phenomena, the thermal and mass diffusions occurring by the simultaneous action of buoyancy forces are of considerable interest in practice. Mixed convection flows, or combined forced and free convection flows, arise in many transport processes in engineering devices and in nature. These flows are characterized by the buoyancy parameter (measure of the influence of the free convection in comparison with that of forced convection on the fluid flow) which depends on the flow configuration and the surface heating conditions.

Ramanaiah *et al*. [1] considered the problem of mixed convection over a horizontal plate subjected to a temperature or surface heat flux varying as a power of x. However, the problem of forced, free and mixed convection flows past a heated or cooled body with porous wall is of interest in relation to the boundary layer control on airfoil, lubrication of ceramic machine parts and food processing. Watanabe [6] has considered the mixed convection boundary layer flow past an isothermal vertical porous flat plate with uniform suction or injection. Sattar [7] made analytical studies on the combined forced and free convection flow in a porous medium. Further, a vast literature of similarity solution has appeared in the area of fluid mechanics, heat transfer, and mass transfer, etc. as it is one of the important means for the reduction of a number of independent variables with simplifying assumptions. Deswita *et al.* [8] obtained a similarity solution for the steady laminar free convection boundary layer flow on a horizontal plate with variable wall temperature. Hossain and Mojumder [9] presented the similarity solution for the steady laminar free convection boundary layer flow generated above a heated horizontal rectangular surface. Furthermore, the study of complete similarity solutions of the unsteady laminar natural convection boundary layer flow above a heated horizontal semi-infinite porous plate have been considered by Hossain *et al.* [10, 11]. Ramanaiah *et al.* [13] studied the similarity solutions of free, mixed and forced convection problems in a saturated porous media. Hossain *et al.* [14] and Md. Hasanuzzan *et al* [17] obtained a complete similarity solution of the unsteady laminar combined free and forced convection boundary layer flow about a heated vertical porous plate in viscous incompressible fluid. The combined free and force convective laminar fluid motion caused by a heated (or cooled) vertical slender body moving through a viscous fluid has not so far been considered for a large scale study. Van Dyke [15] successfully analyzed a natural convection flow near a vertical thin needle for the case of a constant surface temperature. Kuiken [16] has studied the axi-symmetric free convective boundary layer along an isothermal vertical cylinder of constant thickness.

The purpose of the present study is, therefore, to find a possible similarity solution of the combined free and force convective laminar fluid motion caused by a heated (or cooled) axi-symmetric slender body of finite axial length immersed vertically in a viscous incompressible fluid. The thermal distributions on the outer surface of the body as well as the motion of the body itself are assumed to be unsteady. Furthermore, throughout the investigation, the effect of suction or blowing has been taken into consideration. We are attempted to investigate the effects of several involved parameters on the velocity and temperature fields and other flow parameters like skin friction and heat transfer coefficients across the boundary layer. We are also tried to calculate the role of suction or blowing velocity on these parameters as well. The numerical results including the velocity and temperature fields are to be presented graphically for different selected values of the established dimensionless parameters. It is expected that the effects of suction and blowing play an important role on the velocity and temperature fields, so that their effects should be taken into account with other useful parameters associated.

2. Basic Equations of the Flow and Mathematical Analysis

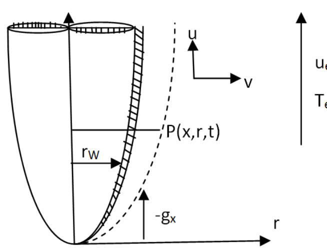

Fig (1). *Physical configuration and coordinate system*

An axi-symmetric heated (or cooled) slender body of finite axial length is immersed vertically in a viscous fluid of variable properties. The surface temperature *(=* T_w *),* the velocity and the temperature of the undisturbed fluid (u_e and T_e) close to the body surface but outside the boundary-layer are all general functions of *x* and *t*. r_w is the radial distance from the axis of symmetry to the surface of the body, *x* is the distance measured along the axis of symmetry of the body and t is the time. The physical configuration and the coordinate system of the problem are shown in Fig.1.

The influence of body force generated by buoyancy effects on the flow field near the surface is significant if the Froude number of such a flow field is of order unity. That is the non dimensional form of the buoyancy force is

$$\frac{T_w - T_e}{T_e}\frac{g_x L_c}{U^2} \cong 0(1) \tag{1}$$

where g_x is the gravity component in the *x*-direction, L_c is a suitable characteristic length and *U* is a suitable characteristic velocity. The non-dimensional form of the equations expressing conservation of mass, momentum and energy for a Boussinesq fluid are follows:

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial r}(rv) = 0 \tag{2}$$

$$\frac{Du}{Dt} = -g_x \beta_T \Delta T \theta + \frac{\partial u_e}{\partial t} + u_e \frac{\partial u_e}{\partial x} + \frac{\nu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) \tag{3}$$

$$\frac{D\theta}{Dt} = -\left\{u\frac{\partial}{\partial x}(\log \Delta T) + \frac{\partial}{\partial t}(\log \Delta T)\right\}\theta + \frac{1}{P_r}\frac{\nu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \theta}{\partial r}\right) \tag{4}$$

Where ρ is the density, *P* is the pressure, μ and λ are the dynamic and second coefficients of viscosity, T the temperature, *k* the thermal conductivity C_p the specific heat at constant pressure and $\beta_T = -\rho^{-1}\left(\frac{\partial \rho}{aT}\right)_P$ the coefficient of cubical expansion of the fluid. $T - T_0 = \Delta T_0$, $\Delta T = T_W - T_0$, *and* $T_e = T_0$ is treated here as constant temperature for the ambient fluid. Since the Boussinesq from of the state relations is $\rho = \rho(T)$, it follows that $\rho_e = \rho_0$ (constant). *T* and T_W in general depend on both *x* and *t*. A solution of the equations (2)-(4) is now sought, these equations being valid in the limit $Re \longrightarrow \infty$ and $E \longrightarrow 0$. Higher order effects are not discussed here, as the present investigation concerns the first order boundary- layer approximations only.

3. Similarity Transforms

The complexity of the above governing differential equations makes the use of simplifying approximations desirable so that tractable solutions may be obtained. The method of similarity provides a convenient and accurate procedure for computing heat transfer, skin friction and other laminar boundary-layer characteristics, Guided by this idea independent variables *(x,r,t)* are changed to a new set of variables (ξ, ϕ, τ) where the relations between the two sets are

$$\xi = x,\ \tau = t,\ \phi = \frac{r^2}{2\gamma(x,\, t)} \tag{5}$$

The continuity equation (2) is identically satisfied by introducing a stream function $\psi(x, r, t)$ defined by $ru = \dfrac{\partial\psi}{\partial r}$, $-rv = \dfrac{\partial\psi}{\partial x}$, we have, $u = 2\dfrac{\partial\psi}{\partial(r^2)} = \dfrac{\partial}{\partial\phi}\left\{\dfrac{\psi}{\gamma(x,t)}\right\}$. Using for the moment, a non-dimensional factor *U (x, t)* for the velocity component *u* is given by $\psi(\xi, \phi, \tau) = \gamma\, U\, F(\xi,\phi,\tau) + \psi(\xi,\phi_0, \tau)$, where $F(\xi,\phi, \tau) = \int_{\phi_0}^{\phi} \dfrac{u}{U} d\phi$ *and* ϕ_0 is the value of ϕ on the body. That is $\phi_0 = \dfrac{r_W^2}{2\gamma(x , t))}$. The velocity components *u* and *v* are found to be $u=UL$, $-rv = (\gamma U F)_\xi - \phi\gamma_\xi U F_\phi - r_W v_W$, where $-r_W v_W = \psi_\xi(\xi,\phi_0,\tau)$ which represents the non-zero wall velocity called suction or blowing velocity normal to the porous surface, so that fluid can either be sucked or blown throughout. Physically, $v_W < 0$ and $v_W > 0$ represent the suction and **blowing** velocity through the porous surface respectively. For uniform suction (or blowing) v_W = constant. However, $v_W \neq 0$ implies that the surface is impermeable to the fluid. In view of the above transformation, equations (2) to (4) become

$$\begin{aligned} &2\nu\{(\varphi+a_3)F_{\varphi\varphi}\}_\varphi + \{a_0(\varphi+a_3)+a_9\}F_{\varphi\varphi} \\ &+(a_1+a_2)F F_{\varphi\varphi} - a_2F_\varphi^2 - a_4F_\varphi + a_5\vartheta + a_6 = 0 \end{aligned} \tag{6}$$

$$\begin{aligned} &\frac{2\nu}{P_r}\{(\varphi+a_3)\vartheta_\varphi\}_\varphi + \{a_0(\varphi+a_3)+a_9\}\vartheta_\varphi \\ &+(a_1+a_2)F\vartheta_\varphi - (a_7 + a_8F_\varphi)\vartheta = 0 \end{aligned} \tag{7}$$

where

$$\begin{aligned} &(i)\ \gamma_\tau = a_0, \\ &(ii)(\gamma UL)_\xi = \gamma_\xi UL + \gamma(UL)_\xi = a_1 + a_2, \\ &(iii)\frac{r_W^2}{2\gamma} = a_3, \\ &(iv)\ \frac{\gamma(UL)_\tau}{UL} = a_4, \\ &(v)\gamma(UL)_\xi = a_2, \\ &(vi)\gamma_\xi(UL) = a_1, \\ &(vii) - \frac{\nu}{UL}g_x\beta_T\Delta T = a_5, \\ &(viii)\frac{\nu}{UL}\{(u_e)_\tau + u_e(u_e)_\xi\} = a_6, \\ &(ix)\ \nu(\log\Delta T)_\tau = a_7, \\ &(x)\ \nu UL\{\log\Delta T\}_\xi = a_8, \\ &(xi) - r_W V_W = a_9\ \ and\ \ \varphi = \frac{r^2 - r_W^2}{2\gamma(x,t)} \end{aligned} \tag{8}$$

The boundary conditions which are imposed in order to determine the solutions of the transformed boundary layer equations (6)-(7) are given by:

$$F(0) = F_\varphi(0) = 0 \;\; F_\varphi(\infty) = \vartheta(0) = 1,\; \vartheta(\infty) = 0 \tag{9}$$

The relations in equation (8) furnish us with the conditions under which similarity solutions are obtained provided that all $a's$ must be constants and thus the equations (6)-(7) will become non-linear ordinary differential equations. In view of the conditions (ii) and (i) stated in equation (8), we have $\gamma = a_0\tau + B(\xi)$ and $\gamma UL = (a_1 + a_2)\xi + A(\tau)$ where $A(\tau)$ is either a function of τ or constant and $B(\xi)$ is a function of ξ or constant. The above two relations one obtains

$$\frac{dA(\tau)}{d\tau}.\frac{dB(\xi)}{d\xi} = a_1(a_0 + a_4) \tag{10}$$

Since *γ(ξ, τ)* and *UL (ξ, τ)* depend wholly on the choice of *A (τ)* and *B (ξ)*, the equation (10) plays a significant role in determining the possible four cases of similarity solutions:

(A)both $\dfrac{dA(\tau)}{d\tau}$ and $\dfrac{dB(\xi)}{d\xi}$ are finite constants,

(B)both $\dfrac{dA(\tau)}{d\tau}$ and $\dfrac{dB(\xi)}{d\xi}$ are zero.

(C) $\dfrac{dA(\tau)}{d\tau} = 0$ but $\dfrac{dB(\xi)}{d\xi} \neq 0$,

(D) $\dfrac{dA(\tau)}{d\tau} \neq 0$ but $\dfrac{dB(\xi)}{d\xi} = 0$

3.1. Similarity Case to Be Considered

Of these four similarity cases, only the Case (A) for which both $dA/d\tau$ and $dB/d\xi$ are finite constants has been studied here. Thus we have $\gamma = a_0\tau + B(\xi)$ and $\gamma UL = (a_1 + a_2)\xi + A(\tau)$. Substituting these in the conditions (i) to (ix) of the equation (8) yields the relations between the constants as follows: $a_0 = a_1 = a_2 = 0$, $a_3 = r_W^2/2B$, a_4 = arbitrary, $a_5 = \dfrac{-g_x\beta_T\Delta TB}{K}e^{\frac{-a_4}{B}\tau}$, $a_6 = a_4$, $a_7 = a_4$, $a_8 = 0$ and a_9 = arbitrary. Substituting the constants and choosing $F = \alpha_1 f$ and $\phi = \alpha_1\eta$ the above equations (6) to (7) reduce to:

$$2(\eta + R_0)f_{\eta\eta\eta} + (2 + F_W)f_{\eta\eta} + 1 - f_\eta + \frac{U_F^2}{u_e^2}\vartheta = 0 \tag{11}$$

$$2(\eta + R_0)\vartheta_{\eta\eta} + (2 + P_r F_W)\vartheta_\eta - P_r\vartheta = 0 \tag{12}$$

The boundary conditions are

$$f(0) = f_\eta(0) = 0\ ,\ f_\eta(\infty) = 1\ ,\vartheta(0) = 1,\ \vartheta(\infty) = 0 \tag{13}$$

where is also chosen $\alpha_1 = \alpha_2$, $U_F^2/u_e^2 = a_5/a_4$, $a_4\alpha_1/\nu = 1$ and writing $r_W^2/2\alpha_1 B = R_0$ with $U_F^2 = -g_x\beta_T\Delta T(Cu_e)$. The terms $U_F^2\vartheta/u_e^2 = a_5\vartheta/a_4$ in the momentum equation indicates how important buoyancy effects are compared with the forced flow effects. The flow is said to be aided when U_F^2/u_e^2 is greater than zero and called an opposing when this parameter is less than zero. When $U_F^2 << u_e^2$ the flow becomes a forced flow, whereas for $U_F^2 << u_e^2$ the flow becomes a free convection flow. The skin friction and heat transfer coefficients τ_w and q_w associated with the equations (11) and (12) are: $\tau_W = \frac{\mu u_e}{\sqrt{C\nu}}\sqrt{2R_0} f_{\eta\eta}(0)$ and $q_W = -\frac{K\Delta T}{\sqrt{C\nu}}\sqrt{2R_0}\,\vartheta_\eta(0)$. The ΔT –variation for this is $\Delta T \infty e^{\frac{t}{C}}$.

4. Numerical Solution and Discussions

The set of differential equations (11)–(12) with the boundary conditions (13) are solved by using computer software. Here the velocity f_η, temperature ϑ are determined as a function of coordinate η. The skin friction coefficient $f_{\eta\eta}(0)$ and the heat transfer rate $-\vartheta_\eta(0)$ are also evaluated for this case and numerical results thus obtained in terms of the similarity variables are displayed in graphs and tables for several selected values of the established parameters F_w, U_F^2/u_e^2, R_0 and Pr below. The effects of F_W on the velocity and temperature fields are plotted in Figure 2 and Figure 3, respectively. From Figures it is observed that, in all cases the velocity is starting at zero, and then velocity increases with the increase of η near the leading edge and finally moves towards 1.0 asymptotically but temperature is starting at 1.0, then it decreases asymptotically and finally leads to zero with the increase of η. From Figure 2 we see that for the case of suction $(F_W > 0)$, the velocity increases with decreasing F_W but for blowing case $(F_W < 0)$, velocity increases with the increase of the magnitude of blowing. The usual stabilizing effect of the suction parameter on the boundary layer growth is also evident from this figure.

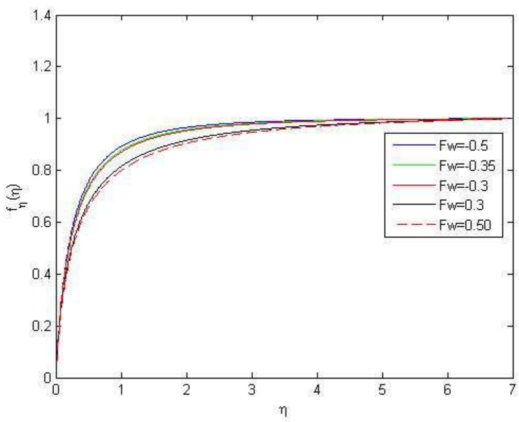

Fig.2

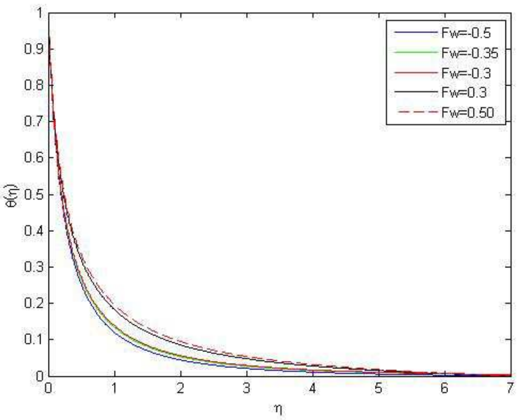

Fig.3

Fig (2). *Velocity profiles and* ***Fig (3).*** *temperature profiles for different values of* F_w *(with fixed values of* $U_F^2/u_e^2 = 0.5$, $R_0 = 0.1$ *and* Pr=0.71*).*

From Figure 3 it is observed that for both the cases of suction and blowing, temperature decreases quickly close to the leading edge and away from it temperature decreases asymptotically and finally leads to zero with the increase of η. For the case of suction $(F_W > 0)$, temperature decreases with decreasing suction. But for the case of blowing $(F_W < 0)$, temperature decreases more with the increase of the magnitude of blowing.

Figure 4 and Figure 5 show the effect of bouncy parameter U_F^2/u_e^2 on the velocity and temperature fields respectively. Physically, the flow is said to be aided when $U_F^2/u_e^2 > 0$ and is called an opposing flow when $U_F^2/u_e^2 < 0$. Further, when $U_F^2 << u_e^2$, the flow becomes forced flow, whereas for $u_e^2 << U_F^2$ the flow becomes a free convection flow. We see from the Figure 4 that with the increase in U_F^2/u_e^2 from negative to positive values, in all cases the velocity is starting at zero and increasing asymptotically to 1.0. But with the increase in U_F^2/u_e^2 the rate of change of velocity increases slightly. Thus before being asymptotically goes to 1.0 far away; velocity is higher for higher values of U_F^2/u_e^2 within the boundary layer. Since the energy equation given by (17) is independent of the buoyancy parameter U_F^2/u_e^2, no effect of this parameter on the temperature field is observed as shown in Figure 5.

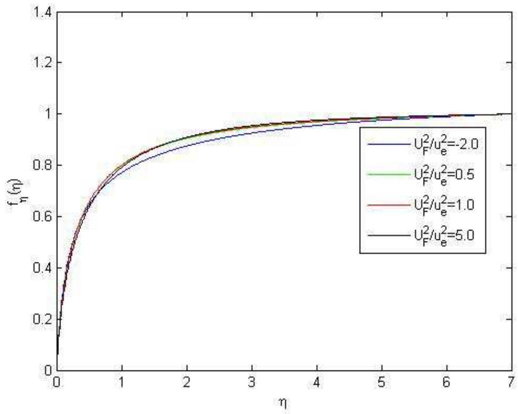

Fig. 4

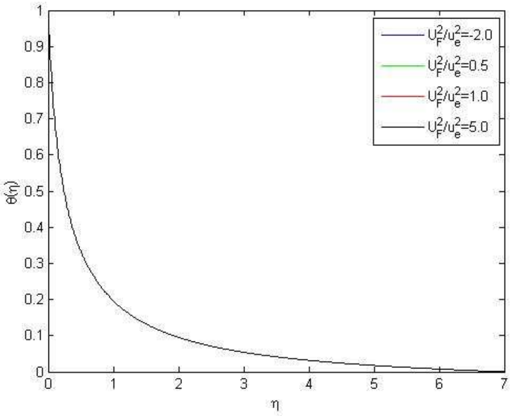

Fig. 5

Fig (4). *Velocity profiles and* ***Fig (5).*** *temperature profiles for different values of* U_F^2/u_e^2 *(with fixed values of* $F_w = 0.5$, $R_0 = 0.1$ *and* $\Pr = 0.71$*).*

The body radius parameter R_0 depends on the shape of the slender body. The velocity and temperature profiles exhibit remarkable changes with the variation of R_0 as observed from the body radius parameter R_0 depends on the shape of the slender body. There is a remarkable consequence of the variation of R_0 on the velocity and temperature fields as are observed in Figure 6 and Figure 7. Like before, for all values of R_0, velocity starts from zero and then increases with the increase of η and finally tends to 1.0, whereas temperature launches from 1.0, then decreases with increasing η and finally asymptotically leads to zero, for large value of η. We see from Figure 6 that within the boundary layer, velocity highly increases with the increase of R_0, before asymptotically being 1.0 for large value of η. From Figure 7 it is observed that temperature decreases sharply with the increase in R_0 before being zero asymptotically for higher value of η.

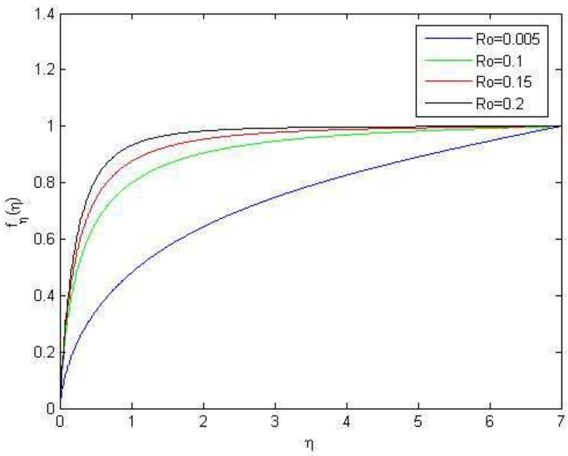

Fig.6

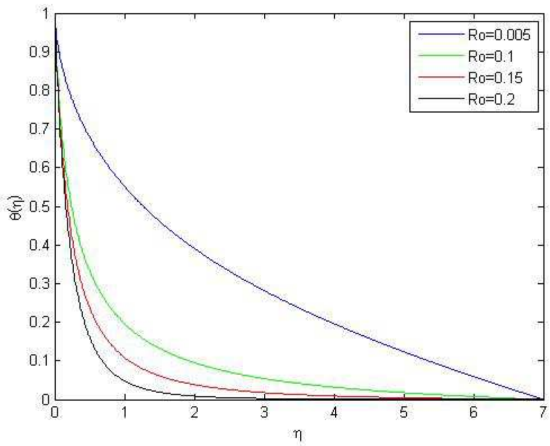

Fig.7

Fig (6). *Velocity profiles and* ***Fig (7).*** *temperature profiles for different values of* R_0 *(with fixed values of* $F_w = .0.5$, $U_F^2/u_e^2 = 0.5$ *and* $\Pr = 0.71$*)*

The last controlling parameter is the Prandtl number $\Pr\left(=\dfrac{\mu C_p}{k}\right)$ which depends on the properties of the medium. Here velocity exhibits minor changes while temperature exhibits significant changes with the variation of Prandtl number $\Pr$. As observed from Figure 8 that, velocity increases negligibly with the increase of $\Pr$ before being 1.0 asymptotically in all cases for large value of η. From Figure 9, we see that, temperature decreases momentously with the decrease in $\Pr$ and asymptotically leads to zero for higher η. Thus before being asymptotically zero temperature is lower for lower values of $\Pr$ within the boundary layer.

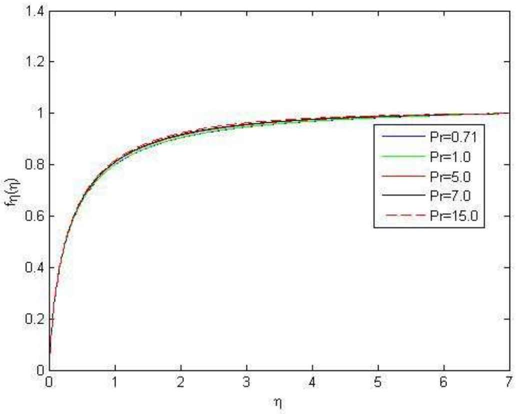

Fig.8

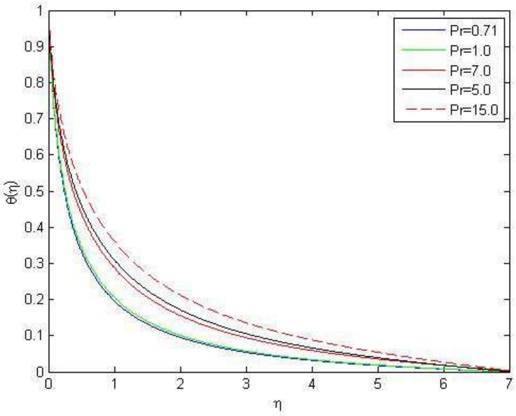

Fig.9

Fig(8). *Velocity profiles and* ***Fig (9).*** *temperature profiles for different values of* Pr *(with fixed values of* $F_w = 0.5$, $U_F^2/u_e^2 = 0.5$ *and* $R_0 = 0.1$*).*

Table 1. *Values proportional to the coefficients of skin-friction (* $f^{//}(0)$ *) and heat transfer (* $-\vartheta^{/}(0)$ *) with the variation of suction parameter* F_w *(for fixed* $U_F^2/u_e^2 = 0.5$, $R_0 = 0.1$ *and* $\Pr = 0.71$*)*

F_w	$f^{//}(0)$	$-\vartheta^{/}(0)$
0.50	0.52341	-1.52677
0.30	0.840835	-2.839873
-0.30	-0.49478	3.80024
-0.50	-0.38129	3.0243

Table 2. *Values proportional to the coefficients of skin-friction (* $f^{//}(0)$ *) and heat transfer (* $-\vartheta^{/}(0)$ *) with the variation of buoyancy parameter* U_F^2/u_e^2 *(for fixed* $F_w = 0.5$, $R_0 = 0.1$ *and* $\Pr = 0.71$*)*

U_F^2/u_e^2	$f^{//}(0)$	$-\vartheta^{/}(0)$
-2.0	0.69423	-1.52677
0.5	0.52341	-1.52677
1.0	0.49312	-1.52677
5.0	0.30218	-1.52677

Table 3. *Values proportional to the coefficients of skin-friction* $f^{//}(0)$ *and heat transfer* $-\vartheta^{/}(0)$ *with the variation of driving parameter* R_0 *(for fixed* $F_w = 0.5$, $U_F^2/u_e^2 = 0.5$ *and* $\Pr = 0.71$.

R_0	$f^{//}(0)$	$-\vartheta^{/}(0)$
0.005	0.52340	-1.52677
0.10	0.52341	-1.62166
0.15	0.51021	-1.63965
0.2	0.50385	-1.70361

Table 4. *Values proportional to the coefficients of skin-friction (* $f^{//}(0)$ *) and heat transfer (* $-\vartheta^{/}(0)$ *) with the variation of Prandtl number* Pr *(for fixed* $F_w = 0.5$, $U_F^2/u_e^2 = 0.5$ *and* $R_0 = 0.1$*)*

Pr	$f^{//}(0)$	$-\vartheta^{/}(0)$
0.71	0.52341	-1.52677
1.00	0.39925	-1.40267
5.00	0.07591	1.50934
7.0	0.04762	1.75093

5. Conclusion

Numerical solution of mixed convective laminar boundary layer flow around a vertical slender body with suction or blowing with the similarity case $\frac{dA}{d\tau}$ and $\frac{dB(\xi)}{d\xi}$ are finite constants has been studied in this paper. On the basis of the findings the following conclusions can be drawn:

(a) Velocity increases with the decrease of suction but for the case of blowing the velocity increases with the increase of the magnitude of blowing. Temperature decreases with decreasing suction but it decreases more with the increase of the magnitude of blowing.

(b) With the increase in the buoyancy parameter U_F^2/u_e^2 from negative to positive values, the velocity enhanced slightly and thus the velocity goes to 1.0 asymptotically within a short range. No effect of buoyancy parameter U_F^2/u_e^2 on the temperature field is observed.

(c) Velocity highly increases with the increase of R_0 and finally leads to 1.0 asymptotically for large η. Temperature decreases sharply with the increase of R_0 and finally approaches zero asymptotically for large η.

(d) With the increase in Pr, the velocity increases negligibly and finally leads to the value 1.0 asymptotically for large value of η .Temperature decreases momentously with decreasing Pr and finally approaches zero asymptotically.

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