

A Detailed Comparison Between Two Methods of Ranking Interval Efficiencies for Fuzzy DEA Models

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Abstract: Data envelopment analysis is a non-parametric technique for measuring and evaluating the relative efficiencies of a set of entities with common inputs and outputs. In fact, in a real evaluation problem input and output data of entities evaluated often fluctuate. This fluctuating data can be represented as linguistic variables characterized by fuzzy numbers for reflecting a kind of general feeling or experience of experts. For this purpose some researchers have proposed several models to deal with the efficiency evaluation problem with the given fuzzy input and output data. One of these methods is to change fuzzy models in to interval models by using alpha cuts. As we may face with some interval efficiency of several entities that should be compare with each other and ranked, in this paper we compare two methods of ranking interval efficiencies that is obtained from interval models. A sensitive difference between these two methods will be shown by a numerical example.

Keywords: Data Envelopment Analysis (DEA), Efficiency, Fuzzy Intervals, Ranking

1. Introduction

Data envelopment analysis (DEA), as a very useful management and decision tool, has found surprising development in theory and methodology and extensive application in the range of the whole world since it was first developed by Charnnes et al. [1]. The term “imprecise data” reflects the situation where some of the inputs and outputs data are only known to lie within bounded intervals (interval numbers), or data that are characterized by fuzzy numbers. Examples include school, hospital, library and most of economic and society systems, in which inputs and outputs are always multiples in character. Actually input and outputs of DMUs are ever-changeable. For example, for evaluating operation efficiencies of airlines, seat-kilometers available, cargo-kilometers available, fuel and labor are regarded as the inputs and passenger-kilometers performed as the output [2]. It is common sense that these inputs and outputs are easy to change because of weather, season, operating state and so on. On the other hand, in more general cases, the data of evaluation are often collected from investigation by polling where the natural language such as *good*, *medium* and *bad* are used to reflect a kind of general situation of the investigated entities rather than a specific case [3, 4]. one of the methods for extending DEA models with fuzzy data is

changing these models in to interval models by using α -cut methods.

In CCR-model with fuzzy parameters, we are faced by two models with two objective functions that are having two optimum solutions [5]. In that of these two models and their solutions, we get an interval efficiency for each DMU as $[\theta^L, \theta^U]$ that is measured from both the optimistic and the pessimistic viewpoints.

2. DEA Models

The α -cut method in fuzzy DEA models convert the model in to two separated models [5].

$$\max \theta_o^L = \sum_{r=1}^s u_r y_{ro}^L$$

$$\text{s.t } \sum_{i=1}^m v_i x_{io}^U = 1$$

$$\sum_{r=1}^s u_r y_{ro}^L - \sum_{i=1}^m v_i x_{io}^U \leq 0$$

$$\sum_{r=1}^s u_r y_{rj}^U - \sum_{i=1}^m v_i x_{ij}^L \leq 0 \quad j = 1, \dots, n, j \neq o$$

$$\begin{aligned}
& u_r \geq 0, r = 1, \dots, s \\
& v_i \geq 0, i = 1, \dots, m \\
& \max \theta_o^U = \sum_{r=1}^s u_r y_{ro}^U \\
& \text{s.t. } \sum_{i=1}^m v_i x_{io}^L = 1 \\
& \sum_{r=1}^s u_r y_{ro}^U - \sum_{i=1}^m v_i x_{io}^L \leq 0 \\
& \sum_{r=1}^s u_r y_{rj}^L - \sum_{i=1}^m v_i x_{ij}^U \leq 0 \quad j = 1, \dots, n, j \neq o \\
& u_r \geq 0, r = 1, \dots, s \\
& v_i \geq 0, i = 1, \dots, m
\end{aligned}$$

In that of the efficiency score for each DMU determined by an interval $[\theta_i^L, \theta_i^U]$, A practical ranking method is needed for comparing and ranking the efficiencies of different DMUs. A few approaches have been developed to rank interval numbers. Here we analysis two methods that are proposed and used for ranking intervals. Although both these methods show extremely the same results, but a sensitive difference will be shown by a numerical example.

3. Methods

3.1. Method 1

Suppose unit i has interval efficiency such as $[\theta_i^L, \theta_i^U]$. First we consider a uniform cumulative distribute for each i and suppose $K = \max \theta_i^U$, then for each unit consider S_i and S_j for two interval data. We can rank the interval efficiency based on the values, $1/S_i, 1/S_j$, as shown in following figures:

1) Figure 1: If $[\theta_i^L, \theta_i^U] \cap [\theta_j^L, \theta_j^U] = \varphi$.

$$1/S_j > 1/S_i \Rightarrow \text{Rank}(DMU_j) > \text{Rank}(DMU_i)$$

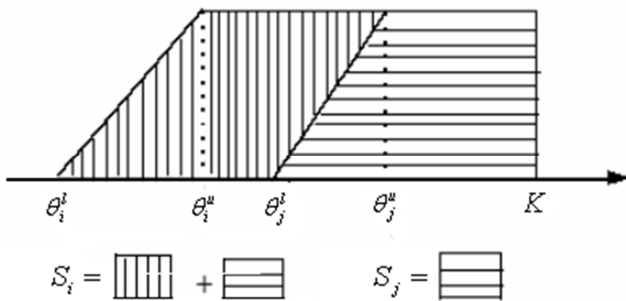


Figure 1. Two intervals with no subscription.

2) Figure 2: If $\theta_i^L < \theta_j^L, \theta_i^L < \theta_j^U, \theta_i^U < \theta_j^U$.

$$1/S_j > 1/S_i \Rightarrow \text{Rank}(DMU_j) > \text{Rank}(DMU_i)$$

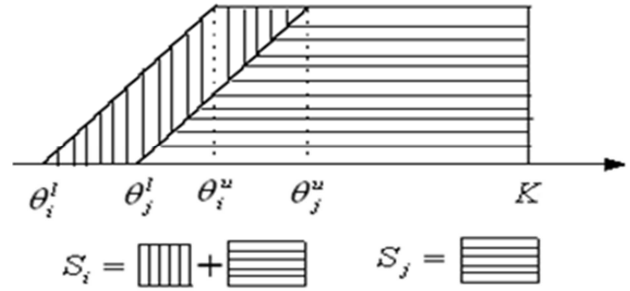


Figure 2. Two overlapped intervals.

3) Figure 3: If $[\theta_i^L, \theta_i^U] \subseteq [\theta_j^L, \theta_j^U]$.

$$\text{if } *_1 < *_2 \Rightarrow S_j \geq S_i \Rightarrow \text{Rank}(DMU_i) \geq \text{Rank}(DMU_j)$$

$$\text{if } *_1 < *_2 \Rightarrow S_i \geq S_j \Rightarrow \text{Rank}(DMU_j) \geq \text{Rank}(DMU_i)$$

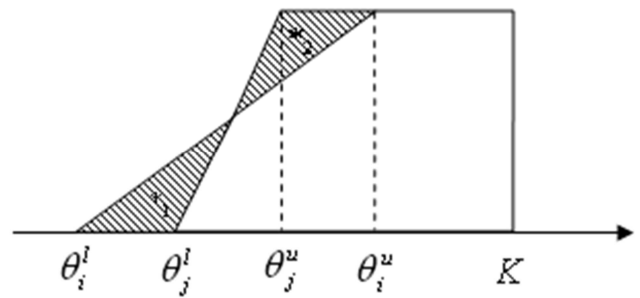


Figure 3. One interval including another.

Note: these ranking methods can be used for n DMUs.

Based on above Interval ranking we can propose another interval ranking that is equivalent with that. If $K = \max \theta_i^U, \forall i$ Then we calculate $R_i = 2K - (\theta_i^L + \theta_i^U)$ for each i , then each DMUs with the lowest R_i has better rank. It means:

$\text{Rank}(DMU_i) \geq \text{Rank}(DMU_j) \Leftrightarrow R_i \leq R_j \Leftrightarrow 2K - (\theta_i^L + \theta_i^U) \leq 2K - (\theta_j^L + \theta_j^U)$. In that of K is a constant value we have: $2K - (\theta_i^L + \theta_i^U) \leq 2K - (\theta_j^L + \theta_j^U) \rightarrow -(\theta_i^L + \theta_i^U) \leq -(\theta_j^L + \theta_j^U) \rightarrow (\theta_i^L + \theta_i^U) \geq (\theta_j^L + \theta_j^U)$. So for each DMU we can only calculate the value $(\theta_i^L + \theta_i^U) \forall i$, and each unit that has more $(\theta_i^L + \theta_i^U)$ value has better rank.

3.2. Method 2

In this approach, developed by Wang et al. [6], some attractive features can be used to compare and rank the efficiency intervals of DMUs. The approach is summarized as follows.

Suppose $\theta_i = [\theta_i^L, \theta_i^U] = \langle m(A_i), w(A_i) \rangle$ ($i = 1, \dots, n$) be the efficiency intervals of n DMUs, where $m(\theta_i) = 1/2 (\theta_i^L + \theta_i^U)$ and $w(\theta_i) = 1/2 (\theta_i^U - \theta_i^L)$ are their midpoints (center) and width. Without loss of generality, suppose $\theta_i = [\theta_i^L, \theta_i^U]$ is chosen as the best efficiency interval. Let $b = \max_{j \neq i} \{\theta_j^U\}$. Obviously, if $\theta_i^L < b$, the DM might suffer the loss of efficiency (also called the loss of opportunity or regret) and feel regret. The maximum loss of efficiency he/she might suffer is given by $\max(r_i) = b -$

$\theta_i^L = \max_{j \neq i} \{\theta_j^U\} - \theta_i^L$. If $\theta_i^L \geq b$, the DM will definitely suffer no loss of efficiency and feel no regret. So the regret of DM is defined to be Zero, i. e. $r_i = 0$. Combining the above two situation, we have $\max(r_i) = \max[\max_{j \neq i} (\theta_j^U) - \theta_i^L, 0]$. Thus, the minimax regret criterion will choose the efficiency interval satisfying the following condition as the best efficiency interval: $\min_i \{\max(r_i)\} = \min_i \{\max[\max_{j \neq i} (\theta_j^U) - \theta_i^L, 0]\}$. Based on the analysis above, we give the following definition for comparing and ranking efficiency intervals.

3.3. Definition

Let $\theta_i = [\theta_i^L, \theta_i^U] = \langle m(A_i), w(A_i) \rangle$ ($i = 1, \dots, n$) be a set of efficiency interval. The maximum loss of efficiency of each efficiency interval θ_i is defined as $R(\theta_i) = \max[\max_{j \neq i} (\theta_j^U) - \theta_i^L, 0] = \max[\max_{j \neq i} \{m(\theta_j) + w(\theta_j)\} - (m(\theta_i) - w(\theta_i)), 0]$. To be able to generate a ranking for a set of efficiency intervals using the maximum losses of efficiency, the following steps are suggested:

3.3.1. Step 1

Calculate the maximum loss of efficiency of each efficiency interval and choose the most desirable efficiency interval that has the smallest maximum loss of efficiency (regret). Suppose θ_{i_1} is selected, where $1 \leq i_1 \leq n$.

3.3.2. Step 2

Eliminate θ_{i_1} from the consideration, recalculate the maximum loss of efficiency of every efficiency interval and determine a most desirable efficiency interval from the remaining $(n - 1)$ efficiency intervals. Suppose θ_{i_2} is chosen, where $1 \leq i_2 \leq n$ but $i_2 \neq i_1$.

3.3.3. Step 3

Eliminate θ_{i_2} from the further consideration, re-compute the maximum loss of efficiency of each efficiency interval and determine a most desirable efficiency interval θ_{i_3} from the remaining $(n - 2)$ efficiency intervals.

3.3.4. Step 4

Repeat the above eliminating process until only one efficiency interval θ_{i_n} is left. The final ranking is $\theta_{i_1} > \theta_{i_2} > \dots > \theta_{i_n}$, where the symbol ($>$) means (is superior to).

3.4. Property 1

Let $A = [a^L, a^U]$ and $B = [b^L, b^U]$ be two intervals. If $a^L \leq b^L$ and $a^U \leq b^U$, then $R(A) > R(B)$. It shows that for two non-nested efficiency intervals, the one with bigger, lower and upper bounds is preferred so the others.

3.5. Property 2

Let $A = [a^L, a^U] = \langle m(A), w(A) \rangle$ and $B = [b^L, b^U] = \langle m(B), w(B) \rangle$ be two efficiency intervals. If A is included in B , i.e. $a^L \geq b^L$ but $a^U \leq b^U$, then

- (1) $R(A) > R(B)$ if $m(A) < m(B)$
- (2) $R(A) = R(B)$ if $m(A) = m(B)$

- (3) $R(A) < R(B)$ if $m(A) > m(B)$

It shows how this method compares and ranks two intervals if one is included in another.

3.6. Property 3

Let $A = [a^L, a^U] = \langle m(A), w(A) \rangle$ and $B = [b^L, b^U] = \langle m(B), w(B) \rangle$ and $C = [c^L, c^U] = \langle m(C), w(C) \rangle$ be three equi-centered efficiency intervals. If $w(A) < w(B) < w(C)$, then $R(A) < R(B)$ and $R(A) < R(C)$. It shows that in equal-centered intervals the interval with the same center but the smallest width is most desirable.

3.7. Numerical Example

Suppose seven DMUs with efficiency intervals in the following table:

Table 1. The efficiency intervals of seven DMUs.

DMU	$[\theta^L, \theta^U]$
A	[0.8088, 1]
B	[0.8593, 1]
C	[0.8764, 0.9829]
D	[0.8100, 0.9205]
E	[0.8062, 0.9097]
F	[0.7499, 0.9016]
G	[0.6007, 0.6713]

In order to compare and rank the efficiency of these seven DMUs, we use both method 1 and method 2. The result shows a sensitive difference that may be important for some decision makers in evaluating DMUs. By using method 2 it is clear that $DMUC$ has the smallest maximum loss of efficiency. So $DMUC$ is rated as the best DMU and eliminated from the further consideration. Again by continuing the algorithm we can see, $DMUB$ has the smallest maximum loss of efficiency. So $DMUB$ is rated as the third best DMUs and eliminated from the further consideration. In the same way we get the ranking order of the seven DMUs as $DMUC > DMUB > DMUA > DMUD > DMUE > DMUF > DMUG$.

(Fig 4 – right graph), by using method 1, the result shows the same as method 2 in 5 DMUs without any difference in ranking $DMUC$ and $DMUB$. So both $DMUB$ and $DMUC$ get the same rank and this Method is not able to distinguish which unit is better than another (Fig4 - left graph).

As we can see in the table, Interval efficiency of $DMUB$ is [0.8593, 1] and Interval efficiency of $DMUC$ is [0.8764, 0.9829]. So by ranking with this Method, we have:

$$0.8593 + 1 = 1.8593$$

$$0.8764 + 0.9829 = 1.8593$$

But by using Method 2 the result shows that:

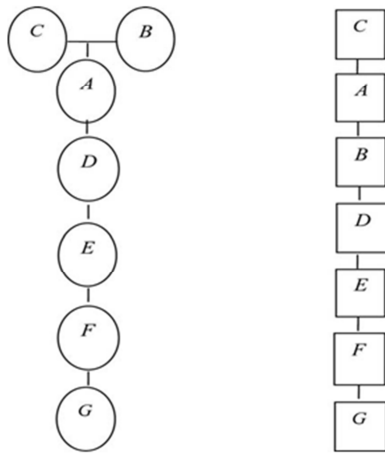
DMUC >> DMUB

Figure 4. Comparing the two mentioned methods.

4. Conclusion

There are several methods for ranking the efficiency of *DMUs* with fuzzy or interval dates and each individual method have its own advantages as well as disadvantages. Therefore comparing these may reach us to the optimum result which can be applicable for decision maker.

References

- [1] T. Entani, Y. Maeda, and H. Tanaka, Dual models of interval DEA and its extension to interval data, *European J. Oper. Res.* 136 (2002) 32-45.
- [2] A. Charnes, W. W. cooper, and T. Sueyoshi, Least squares/ridge regression and goal programming/constrained regression alternatives, *European J. Oper. Res.* 88 (1996) 525-536.
- [3] M. Tavana, R. Khanjani Shiraz, A. Hatami-Marbini, P. J. Agrell, Kh. Paryab, Chance-constrained DEA models with random fuzzy inputs and outputs, *Knowledge-Based Systems*, Volume 52, November 2013, Pages 32-52.
- [4] P. Guo, H. Tanaka, Extended fuzzy DEA, *Proc. 3rd Asian Fuzzy Systems Symp.*, 1998, pp. 517-521.
- [5] Shiang-Tai Liu, Restricting weight flexibility in fuzzy two-stage DEA, *Computers & Industrial Engineering*, Volume 74, August 2014, Pages 149-160.
- [6] F. Nagano, T. Yamaguchi, and T. Fukukawa, DEA with fuzzy output data, *J. Oper. Res. Soc. Jpn.* 40 (1995) 425-429.
- [7] Y. M. Wang, J. B. Yang, and D. L. Xu, Two approaches for ranking interval numbers based on decision making under uncertainty, *Decis. Support System*, submitted for publication.
- [8] Peijum Gue, and H. Tanaka, Fuzzy DEA: a perceptual evaluation method, *Fuzzy sets and systems* 119 (2001) 149-160.
- [9] M. Esmaili, An Enhanced Russell Measure in DEA with interval data, *Applied Mathematics and Computation*, Volume 219, Issue 4, 1 November 2012, Pages 1589-1593.
- [10] H. Li, W. Yang, Zh. Zhou and Ch. Huang, Resource allocation models' construction for the reduction of undesirable outputs based on DEA methods, *Mathematical and Computer Modelling*, Volume 58, Issues 5-6, September 2013, Pages 913-926.
- [11] M. Toloo, E. Keshavarz, A New Two-phase Approach for Holding the Strict Positivity Restriction of Weights in DEA Models, *Procedia Economics and Finance*, Volume 26, 2015, Pages 575-583.
- [12] P. T. Chang, J. H. Lee, A fuzzy DEA and knapsack formulation integrated model for project selection, *Computers & Operations Research*, Volume 39, Issue 1, January 2012, Pages 112-125.
- [13] J. Puri, S. P. Yadav, A concept of fuzzy input mix-efficiency in fuzzy DEA and its application in banking sector, *Expert Systems with Applications* 40 (2013) 1437-1450.
- [14] A. Charnes, W. W. Cooper, and E. Rhodes, Measuring the efficiency of decision making units, *European J. Oper. Res.* 2 (1978) 429-444.