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New Predictor-Corrector Iterative Methods with Twelfth-Order Convergence for Solving Nonlinear Equations

Noori Yasir Abdul-Hassan

Department of Mathematics, College of Education for Pure Sciences, University of Basrah, Basrah, Iraq

Email address:
noori.math86@gmail.com

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Abstract: In this paper, we propose and analyze new two efficient iterative methods for finding the simple roots of nonlinear equations. These methods based on a Jarratt's method, Householder's method and Chun&Kim's method by using a predictor-corrector technique. The error equations are given theoretically to show that the proposed methods have twelfth-order convergence. Several numerical examples are given to illustrate the efficiency and robustness of the proposed methods. Comparison with other well-known iterative methods is made.

Keywords: Nonlinear Equations, Predictor-Corrector Methods, Convergence Analysis, Efficiency Index, Numerical Examples

1. Introduction

The problem of solving a single nonlinear equation f(x) = 0 is fundamental in various branches of science and engineering. Recently, there are many numerical iterative methods have been developed to solve these problems. These methods are constructed by using several different techniques, such as Taylor series, quadrature formulas, homotopy perturbation technique and its variant forms, decomposition technique, variational iteration technique, and Predictor-corrector technique. For more details, see [1-4, 6-32]. In this paper, we use the Predictor-corrector technique to construct some new iterative methods based on a Jarratt's method as a predictor with Householder's method and Chun &Kim's method as a correctors. The orders of convergence and corresponding error equations of the obtained iteration formulae are derived analytically to show that our proposed methods have twelfth -order convergences. Each one of these methods requires two evaluations of the function, three evaluations of first-derivative and one evaluations of second-derivative per iteration. Therefore, our proposed methods have the same efficiency index is $12^{1/6} \cong 1.51309$. To illustrate the performance of these new methods, we give several examples and a comparison with other well-known iterative methods is given.

2. Preliminaries

Definition 2.1 (see [12, 32]): Let $\alpha \in R$, $x_n \in R$, $n = 0, 1, 2, \ldots$. Then the sequence $\{x_n\}$ is said to converge to α if $\lim_{n\to\infty}|x_n - \alpha| = 0$. If, in addition, there exist a constant $c \geq 0$, an integer $n_0 \geq 0$ and $p \geq 0$ such that for all $n > n_0$,
$x_{n+1} - \alpha \ | \leq c\ |x_n - \alpha|^p$, then $\{x_n\}$ is said to be convergence to α with convergence order at least p. If $p = 2$ or 3, the convergence is said to be quadratic or cubic respectively.

Notation 2.1: Let $e_n = x_n - \alpha$ is the error in the n^{th} iteration. Then the relation

$$e_{n+1} = c\, e_n^p + o(e_n^{p+1}) \tag{2.1}$$

is called the *error equation* for the method. By substituting $e_n = x_n - \alpha$ for all n in any iterative method and simplifying, we obtain the error equation for that method. The value of p obtained is called *order of convergence* of this method which produces the sequence $\{x_n\}$.

Definition 2.2 (see [3, 6]): *Efficiency index* is simply defined as

$$E.I.=p^{1/m} \tag{2.2}$$

Where p is the order of the method and m is the number of functions evaluations required by the method (units of work periteration).

3. Construction of the Method

In this section, we recall some of the important methods such as Newton's method, Jarratt's method, Halley's method, Householder's method Chun & Kim's method and Jarratt-Halley's method in the following six Algorithms:

Algorithm (3.1): For a given x_0, compute approximates solution x_{n+1} by the iterative scheme:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{3.1}$$

This is the well-known Newton's method, which has a quadratic convergence [5]. Its efficiency is 1.41421.

Algorithm (3.2): For a given x_0, compute approximates solution x_{n+1} by the iterative scheme:

$$x_{n+1} = x_n - J_{f(x_n)} \frac{f(x_n)}{f'(x_n)} \tag{3.2}$$

Where $J_{f(x_n)} = \frac{3f'(y_n)+f'(x_n)}{6f'(y_n)-2f'(x_n)}$ and $y_n = x_n - \frac{2f(x_n)}{3f'(x_n)}$. This is known as Jarratt's fourth-order method [4, 8]. Its efficiency is 1.58740.

Algorithm (3.3): For a given x_0, compute approximates solution x_{n+1} by the iterative scheme:

$$x_{n+1} = x_n - \frac{2f(x_n)f'(x_n)}{2f'^2(x_n)-f(x_n)f''(x_n)} \tag{3.3}$$

This is known as Halley's method [9, 10, 11, 13, 23], which has cubic convergence and its efficiency is 1.44225.

Algorithm (3.4): For a given x_0, compute approximates solution x_{n+1} by the iterative scheme:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}\{ 1 + \frac{f(x_n)f''(x_n)}{2f'^2(x_n)}\} \tag{3.4}$$

This is known as Householder's method, which has cubic convergence [21]. Its efficiency is 1.44225.

Algorithm (3.5): For a given x_0, compute approximates solution x_{n+1} by the iterative scheme:

$$x_{n+1} = x_n - \frac{f(x_n)f'(x_n)\{2+2f'^2(x_n)+f(x_n)f''(x_n)\}}{2f'^2(x_n)\{1+f'^2(x_n)\}-f(x_n)f''(x_n)} \tag{3.5}$$

This is the third order method was referred by Chun and Kim [6, 14]. Its efficiency is 1.44225.

Algorithm (3.6): For a given x_0, compute approximates solution x_{n+1} by the iterative scheme:

$$y_n = x_n - \frac{2f(x_n)}{3f'(x_n)} \tag{3.6.a}$$

$$z_n = x_n - J_{f(x_n)} \frac{f(x_n)}{f'(x_n)} \tag{3.6.b}$$

$$x_{n+1} = z_n - \frac{2f(z_n)f'(z_n)}{2f'^2(z_n)-f(z_n)f''(z_n)} \tag{3.6.c}$$

Where $J_{f(x_n)} = \frac{3f'(y_n)+f'(x_n)}{6f'(y_n)-2f'(x_n)}$. This is known as Jarratt-Haley's method [2], which has twelfth-order of convergence. Its efficiency is 1.513086.

Now, we present the following new two predictor-corrector iterative methods which have twelfth-order convergence, based on a combination scheme between Jarratt's method and each one of Householder's method and Chun&Kim's method, by using Algorithm (3.2) as a predictor and Algorithm (3.4) and Algorithm (3.5) as a corrector, for solving the nonlinear equation $f(x)=0$.

Algorithm (3.7): For a given x_0, compute approximates solution x_{n+1} by the iterative scheme:

$$y_n = x_n - \frac{2f(x_n)}{3f'(x_n)} \tag{3.7.a}$$

$$z_n = x_n - \{\frac{3f'(y_n)+f'(x_n)}{6f'(y_n)-2f'(x_n)}\} \frac{f(x_n)}{f'(x_n)} \tag{3.7.b}$$

$$x_{n+1} = z_n - \frac{f(z_n)}{f'(z_n)}\{ 1 + \frac{f(z_n)f''(z_n)}{2f'^2(z_n)}\} \tag{3.7.c}$$

This is called the predictor-corrector Jarratt-Householder's method (JHHM), which has twelfth-order of convergence. Its efficiency is 1.513086.

Algorithm (3.8): For a given x_0, compute approximates solution x_{n+1} by the iterative scheme:

$$y_n = x_n - \frac{2f(x_n)}{3f'(x_n)} \tag{3.8.a}$$

$$z_n = x_n - \{\frac{3f'(y_n)+f'(x_n)}{6f'(y_n)-2f'(x_n)}\} \frac{f(x_n)}{f'(x_n)} \tag{3.8.b}$$

$$x_{n+1} = z_n - \frac{f(z_n)f'(z_n)\{2+2f'^2(z_n)+f(z_n)f''(z_n)\}}{2f'^2(z_n)\{1+f'^2(z_n)\}-f(z_n)f''(z_n)} \tag{3.8.c}$$

This is called the predictor-corrector Jarratt-Chun&Kim's method (JCKM), which has twelfth-order of convergence. Its efficiency is 1.513086.

4. Convergence Analysis of the Methods

In this section, we compute the orders of convergence and corresponding error equations of the proposed methods (Algorithm (3.7) and Algorithm (3.8)) as follow.

Theorem 4.1: Let $\alpha \in I$ be a simple zero of sufficiently differentiable function $f: I \subseteq R \to R$ for an open interval I. If x_0 is sufficiently close to α, then the iterative method defined by Algorithm (3.7) is of order twelve and it satisfies the following error equation:

$$e_{n+1}= (c_3^4c_2^3 - 5c_3^3c_2^5 + 9c_3^2c_2^7 - (1/729)c_3c_4^3 - 7c_3c_2^9 + (2/729)c_4^3c_2^2 + (2/27)c_4^2c_2^5 + (2/3)c_4c_2^8 + 2c_2^{11} - (1/9)c_3c_4^2c_2^3 - (5/3)c_3c_4c_2^6 + (1/27)c_4^2c_2c_3^2 - (1/3)c_4c_3^3c_2^2 + (4/3)c_4c_3^2c_2^4)e^{12} + O(e^{13}) \tag{4.1}$$

Where $\boldsymbol{c_k} = \frac{f^{(k)}(\alpha)}{f'(\alpha)}$, k=2, 3, …, and

$$e_n = x_n - \alpha \tag{4.2}$$

Proof: Let α be a simple zero of f. Then by expanding $f(x_n)$ and $f'(x_n)$ in Taylor's series about x = α, we get

$$f(x_n) = f'(\alpha)\{e + c_2e^2 + c_3e^3 + c_4e^4 + c_5e^5 + \ldots\} \tag{4.3}$$

$$f'(x_n) = f'(\alpha)\{1 + 2c_2e + 3c_3e^2 + 4c_4e^3 + 5c_5e^4 + 6c_6e^5 + \ldots\} \tag{4.4}$$

From (4.3) and (4.4), we have

$$\frac{f(x_n)}{f'(x_n)} = e - c_2e^2 + (-2c_3 + 2c_2^2)e^3 + (7c_2c_3 - 3c_4 - 4c_2^3)e^4 + (10c_2c_4 - 4c_5 + 6c_3^2 - 20c_3c_2^2 + 8c_2^4)e^5 + \tag{4.5}$$

Also,

$$\frac{2f(x_n)}{3f'(x_n)} = (2/3)e - (2/3)c_2e^2 + (-(4/3)c_3 + (4/3)c_2^2)e^3 + ((14/3)c_2c_3 - 2c_4 - (8/3)c_2^3)e^4 + ((20/3)c_2c_4 - (8/3)c_5$$
$$+ 4c_3^2 - (40/3)c_3c_2^2 + (16/3)c_2^4)e^5 + \ldots \tag{4.6}$$

Substituting (4.6) and (4.2) into (3.7.a), and simplifying, we have

$$y_n = \alpha + (1/3)e + (2/3)c_2e^2 + ((4/3)c_3 - (4/3)c_2^2)e^3 + (-(14/3)c_2c_3 + 2c_4 + (8/3)c_2^3)e^4 + (-(20/3)c_2c_4$$
$$+ (8/3)c_5 - 4c_3^2 + (40/3)c_3c_2^2 - (16/3)c_2^4)e^5 + \ldots \tag{4.7}$$

From (4.7), using Taylor's expansion and simplifying, we have

$$f'(y_n) = f'(\alpha)\{1 + (2/3)c_2e + ((4/3)c_2^2 + (1/3)c_3)e^2 + (4c_2c_3 + (4/27)c_4 - (8/3)c_2^3)e^3 + ((44/9)c_2c_4 - 32/3)c_3c_2^2$$
$$+ (8/3)c_3^2 + (16/3)c_2^4)e^4 + ((52/9)c_4c_3 - (40/3)c_4c_2^2 + (16/3)c_2c_5 - 12c_2c_3^2 + (80/3)c_3c_2^3 - (32/3)c_2^5)e^5 + \ldots\} \tag{4.8}$$

Combining (4.8) and (4.4), using Taylor's expansion and simplifying, we have

$$3f'(y_n) + 2f'(x_n) = 4 + 4c_2e + (4c_2^2 + 4c_3)e^2 + ((40/9)c_4 + 12c_2c_3 8c_2^3)e^3 + (5c_5 + (44/3)c_2c_4 - 32c_3c_2^2 8c_3^2 + 16c_2^4)e^4$$
$$+ ((52/3)c_4c_3 - 40c_4c_2^2 + 16c_2c_5 - 36c_2c_3^2 + 80c_3c_2^3 - 32c_2^5 + 6c_6)e^5 + \ldots \tag{4.9}$$

Also,

$$6f'(y_n) - 2f'(x_n) = 4 + (8c_2^2 - 4c_3)e^2 + (-(64/9)c_4 + 24c_2c_3 - 16c_2^3)e^3 + (-10c_5 + (88/3)c_2c_4 - 64c_3c_2^2 + 16c_3^2 + 32c_2^4)e^4$$
$$+ ((104/3)c_4c_3 - 80c_4c_2^2 + 32c_2c_5 - 72c_2c_3^2 + 160c_3c_2^3 - 64c_2^5 - 12c_6)e^5 + \ldots \tag{4.10}$$

Dividing (4.9) by (4.10), using Taylor's expansion and simplifying, we have

$$\frac{3f'(y_n) + 2f'(x_n)}{6f'(y_n) - 2f'(x_n)} = 1 + c_2e + (-c_2^2 + 2c_3)e^2 + ((26/9)c_4 - 2c_2c_3)e^3 + (-(17/9)c_2c_4 - 3c_3c_2^2 + 2c_2^4 + (15/4)c_5)e^4$$
$$+ (-(3/2)c_2c_5 - (44/9)c_4c_2^2 + 14c_3c_2^3 - 9c_2c_3^2 - 4c_2^5 + (19/9)c_4c_3 + (9/2)c_6)e^5 + \ldots \tag{4.11}$$

Combining (4.5) and (4.11), using Taylor's expansion and simplifying, we have

$$\left\{\frac{3f'(y_n) + 2f'(x_n)}{6f'(y_n) - 2f'(x_n)}\right\}\frac{f(x_n)}{f'(x_n)} = e + (c_2c_3 - c_2^3 - (1/9)c_4)e^4 + ((20/9)c_2c_4 - (1/4)c_5 + 2c_3^2 - 8c_3c_2^2 + 4c_2^4)e^5 + \ldots \tag{4.12}$$

Substituting (4.12) and (4.2) into (3.7.b), and simplifying, we have

$$z_n = \alpha + (-c_2c_3 + c_2^3 + (1/9)c_4)e^4 + (-(20/9)c_2c_4 + (1/4)c_5 - 2c_3^2 + 8c_3c_2^2 - 4c_2^4)e^5 + \ldots \tag{4.13}$$

From (4.13), using Taylor's expansion and simplifying, we have

$$f(z_n) = f'(\alpha)\{(-c_2c_3 + c_2^3 + (1/9)c_4)e^4 + (-(20/9)c_2c_4 + (1/4)c_5 - 2c_3^2 + 8c_3c_2^2 - 4c_2^4)e^5 + \ldots\} \tag{4.14}$$

And,

$$f'(z_n) = f'(\alpha)\{1 + (-2c_2^2c_3 + 2c_2^4 + (2/9)c_2c_4)e^4 - ((40/9)c_2^2c_4 - (1/2)c_2c_5 + 4c_2c_3^2 - 288c_3c_2^3 + 144c_2^5)e^5 + \ldots\} \tag{4.15}$$

Also,

$$f''(z_n)= f'(\alpha)\{2c_2+ (-6c_2c_3^2+6c_3c_2^3+ (2/3)\, c_3c_4)\, e^4- ((40/3)\, c_2c_3 c_4-(3/2)\, c_3c_5+12c_3^3-48c_3^2c_2^2+ 24c_3c_2^4)\, e^5+ \dots\} \quad (4.16)$$

From (4.14), (4.15) and (4.16), using Taylor's expansion and simplifying, we have

$$\frac{f(z_n)}{f'(z_n)} = (-c_2c_3+ c_2^3+(1/9)\, c_4)\, e^4+ (-(20/9)\, c_2c_4+ (1/4)\, c_5- 2c_3^2+ 8c_3c_2^2- 4c_2^4)\, e^5+ \dots \quad (4.17)$$

And,

$$1+ \frac{f(z_n)\, f''(z_n)}{2[f'(z_n)]^2}= 1+ (-c_2^2c_3+c_2^4+(1/9)\, c_2c_4)\, e^4- ((20/9)\, c_2^2c_4- (1/4)\, c_2c_5+ 2c_2c_3^2- 8c_3c_2^3+ 4c_2^5)\, e^5+ \dots \quad (4.18)$$

Combining (4.17) and (4.18), using Taylor's expansion and simplifying, we have

$$\frac{f(z_n)}{f'(z_n)}\{1 + \frac{f(z_n)\, f''(z_n)}{2[f'(z_n)]^2}\}= (-c_2c_3+c_2^3+ (1/9)\, c_4)\, e^4+ (-(20/9)\, c_2c_4+ (1/4)\, c_5- 2c_3^2+ 8c_3c_2^2- 4c_2^4)\, e^5+ \dots \quad (4.19)$$

Thus, substituting (4.13) and (4.19) into (3.7.c), using Taylor's expansion and simplifying, we have

$$x_{n+1} = \alpha+ (c_3^4c_2^3- 5c_3^3c_2^5+ 9c_3^2c_2^7- (1/729)c_3c_4^3- 7c_3c_2^9+ (2/729)c_4^3c_2^2+ (2/27)c_4^2c_2^5+ (2/3)c_4c_2^8 +2c_2^{11}$$
$$-(1/9)c_3c_4^2c_2^3- (5/3)c_3c_4c_2^6+ (1/27)c_4^2c_2c_3^2- (1/3)c_4c_3^3c_2^2+ (4/3)c_4c_3^2c_2^4)e^{12}+ O(e^{13}) \quad (4.20)$$

Which implies that

$$e_{n+1} = (c_3^4c_2^3- 5c_3^3c_2^5+ 9c_3^2c_2^7- (1/729)c_3c_4^3- 7c_3c_2^9+ (2/729)c_4^3c_2^2+ (2/27)c_4^2c_2^5+ (2/3)c_4c_2^8+ 2c_2^{11}$$
$$- (1/9)c_3c_4^2c_2^3- (5/3)c_3c_4c_2^6+ (1/27)c_4^2c_2c_3^2- (1/3)c_4c_3^3c_2^2+ (4/3)c_4c_3^2c_2^4)e^{12}+ O(e^{13}) \quad (4.21)$$

This is show that Algorithm (3.7) is twelve-order convergent.

Theorem 4.2: Let $\alpha \in I$ be a simple zero of sufficiently differentiable function $f: I \subseteq R \to R$ for an open interval *I*. If x_0 is sufficiently close to α, then the iterative method defined by Algorithm (3.8) is of order twelve and it satisfies the following error equation:

$$e_{n+1}= (c_3^4c_2^3- (9/2)c_3^3c_2^5+ (15/2)c_3^2c_2^7- (1/729)c_3c_4^3- (11/2)c_3c_2^9+ (1/486)c_4^3c_2^2+ (1/18)c_4^2c_2^5+ (1/2)c_4c_2^8$$
$$+ (3/2)c_2^{11}+ (1/27)c_4^2c_2c_3^2- (1/3)c_4c_3^3c_2^2+ (7/6)c_4c_3^2c_2^4-(5/54)c_3c_4^2c_2^3- (4/3)c_3c_4c_2^6)e^{12}+ O(e^{13}) \quad (4.22)$$

Proof. Similar procedure, to the proof of theorem 4.1, can be applied to analyze the convergence of Algorithm (3.8).

5. Numerical Examples

In this section, we present the results of numerical calculations on different functions and initial points to demonstrate the efficiency of proposed methods, Jarratt-Householder's method (JHHM) and Jarratt- Chun&Kim's method (JCKM). Also, we compare these methods with the classical Newton's method (NM) and other methods, as Jarratt's method (JM), Halley's method (HM), Householder's method (HHM), Chun&Kim's method (CKM), and Jarratt-Halley's method (JHM). All computations are carried out with double arithmetic precision. We use the stopping criteria $|x_{n+1} - x_n| < \epsilon$ and $|f(x_{n+1})| < \epsilon$, where $\epsilon = 10^{-15}$, for computer programs. All programs are written in MATLAB.

Different test functions and their approximate zeros x^* found up to the 15^{th} decimal place are given in Table 1, the efficiency index (*E.I.*) of various iterative methods is given in Table 2 and the number of iterations (*NITER*) to find x^* is given in Table 3. NC in Table 3 means that the method does not converge to the root x^*.

Table 1. *Different test functions and their approximate zeros (x^*).*

Functions	x^*
$f_1(x)= x^3+4x^2-1$	1.365230013414097
$f_2(x)= \sin(x)-x/2$	1.895494267033981
$f_3(x)= e^{-x}+ \cos(x)$	1.746139530408012
$f_4(x)=e^x\sin(x)+ \ln(x^2+1)$	0
$f_5(x)= (x-2)^{23}- 1$	3
$f_6(x)= x\, e^{x^2} – \sin^2(x)+ 3\cos(x)+5$	-1.207647827130919
$f_7(x)= \sin^{-1}(x^2-1)- x/2+1$	0.5948109683983692

Table 2. *Comparisons between the methods depending on the efficiency index (E.I.).*

	NM	JM	HM	HHM	JHM	JHHM	JCKM
p	2	4	3	3	12	12	12
m	2	3	3	3	6	6	6
E.I.= $p^{1/m}$	1.414214	1.587401	1.442250	1.442250	1.513086	1.513086	1.513086

Table 3. *Comparisons between the methods depending on the number of iterations (NITER).*

Functions	Initial points x_0	Number of Iterations(NITER)							
		NM	JM	HM	CKM	HHM	JHM	JCKM	JHHM
	-0.3	54	39	52	7	10	14	6	11
$f_1(x)$	-0.01	29	26	39	6	27	11	9	8
	0.8	6	3	4	4	5	2	2	2
	-1	14	4	6	6	NC	3	3	3
$f_2(x)$	1.6	6	3	3	3	4	2	2	2
	6	17	16	11	8	4	5	4	3
	-0.3	5	3	8	8	8	2	3	2
$f_3(x)$	2	4	2	6	7	NC	2	2	2
	3	7	4	23	15	NC	3	3	3
	-0.1	5	3	3	3	5	2	2	2
$f_4(x)$	2.3	9	5	7	7	6	3	3	3
	3.5	6	4	4	4	4	3	3	3
	0.2	713	66	48	52	53	83	28	19
$f_5(x)$	2.9	12	5	4	NC	NC	3	3	3
	5	30	13	NC	16	16	77	8	8
	-1.2	4	3	3	3	3	2	2	2
$f_6(x)$	1.2	623	9	14	5	5	NC	8	8
	-3	15	7	8	NC	10	5	5	5
	-0.9	4	3	3	3	3	2	2	2
$f_7(x)$	0.85	4	2	3	3	3	2	2	2
	1	5	3	3	3	3	2	2	2

6. Conclusion

In this paper, we presented new two predictor-corrector iterative methods with twelfth-order convergence for solving nonlinear equations, which are based on the Jarratt's method, Householder's method and Chun & Kim's method. The proposed methods have the same efficiency index is equal to 1.513086. From numerical experiments we show that our methods are efficient, robust and faster convergence in comparison with classical Newton's method and some other methods.

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