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Heat Transfer Analysis over the Boundary Layer Stagnation-Point Flow of Couple Stress Fluid over an Exponentially Stretching Sheet

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Abstract: The present study provides a comprehensive discussion for the problem of boundary layer flow and heat transfer analysis of stagnation point flow of couple stress fluid over an exponentially stretching surface. The governing equations of couple stress fluid model are assumed under boundary layer approach. The nonlinear partial differential equations are simplified by using similar similarity transformations. The analytical solutions of abridged equations are computed with the help of homotopy analysis method (HAM). The convergence of the HAM solutions have been deliberated by plotting $\hbar$ - curves and also through homotopy pade approximation. The physical features of pertinent parameters have been discussed through graphs.

Keywords: Boundary Layer Flow, Couple Stress Fluid, Exponential Stretching/Shrinking, Analytical Solution

1. Introduction

The study of flow and heat transfer of steady and unsteady, compressible and incompressible, viscous and non-Newtonian fluids over stretching surfaces have tremendous applications in several physical situations such as cooling of metallic plates, extrusion of polymers, aerodynamic extrusion of plastic sheets, purification of liquefied metal from non-metallic inclusion and in manufacturing process of artificial films and fibers etc. Nazar et al. [1] have discussed the time dependent boundary layer flow of a non-Newtonian micropolar fluid over a stretching sheet, where the sheet was assumed to be stretched in its own plane. The porosity effects over stagnation flow and heat transfer of viscous fluid over a stretching/shrinking sheet immersed in a saturated medium has been tackled be Rosalie et al. [2]. Further, Nadeem and Awis [3] have examined the effects of temperature dependent viscosity and thermo-capillarity on the flow and heat transfer of a viscous fluid in a thin film on a horizontal porous shrinking sheet through a porous medium. The suction/injection effects on flow and heat transfer of a viscous fluid on a stretching sheet in a porous medium with internal heat generation or absorption was studied by Cortell [4]. Furthermore, Chiam [5] studied the problem of boundary layer flow and heat transfer of an electrically conducting fluid over a non-isothermal stretching sheet under the influence of a transverse magnetic field. Few other important works concerning the boundary layer flow and heat transfer of viscous and non-Newtonian fluids over stretching sheet are included in [6-10].

Recently, couple stress fluid model has been given a lot of consideration by numerous researchers in diverse physical circumstances. Lin [11] has inspected the linear stability analysis of rotor-bearing organism with couple stress fluid as lubricants. Zakaria [12] has measured the effects of magnetohydrodynamic over the problem of unsteady free convection flow of a couple stress fluid with a relaxation time through porous medium. Very recently, Nadeem and Akram [13] have estimated the induced magnetic field effects over the peristaltic flow of a couple stress fluid in an asymmetric channel. In another work, Lin [14] has encountered the problem of squeeze film characteristics of finite journal bearings in a couple stress fluid model. Later on, Ogulu [15] has examined the problem of oscillating plate temperature flow of a polar fluid past a vertical porous plate in the presence of couple stresses and radiation. The available work concerning the couple stress fluids are mostly in peristaltic flow under long wavelength approximations and couple stress fluid flow through finite geometry but not much work is available about the couple stress fluid in case of the boundary layer flow for infinite geometry.

The determination of the present work is to scrutinize the problem of stagnation-point boundary layer flow and heat transfer of a couple stress fluid over an exponentially stretching surface. The governing equations of the problem are first condensed using the boundary layer theory and are then simplified using similar similarity variables [16-21]. The resulting system of differential equations is finally solved analytically through homotopy analysis method (HAM). The convergence of the HAM solutions is presented through graphs and also through homotopy pade approximations [22-26]. Further physical insight is also presented at the end.

2. Formulation

Let us consider the boundary layer stagnation point flow of a steady incompressible couple stress fluid streaming over an exponentially stretching surface. The coordinates (x,y) are chosen such that x is taken along the surface of the sheet, while y is assumed normal to it. The nonlinear boundary layer equations of conservation of mass, momentum and heat transfer [27-31] in absence of dissipation are

$$u_x + v_y = 0, \tag{1}$$

$$uu_x + vu_y = U_\infty \frac{dU_\infty}{dx} + \upsilon u_{yy} - \frac{\eta_0}{\rho} u_{yyyy}, \tag{2}$$

$$uT_x + vT_y = \alpha T_{yy}, \tag{3}$$

here (u,v) are the velocity components along the (x,y) axes, υ is the kinematic viscosity, U_∞ is the free-stream velocity, η_0 is the material constant for the couple stress fluid ρ is the density, T is temperature and α is the thermal diffusivity. The corresponding boundary conditions for the problem are

$$u = U_w, \quad v = 0, \ \text{at}\ y = 0, \tag{4}$$

$$u \to U_\infty, \ \text{as} \ y \to \infty, \tag{5}$$

$$\frac{d^2u}{dy^2} = 0, \quad \frac{d^2v}{dy^2} = 0, \quad \text{for}\ y = 0, \ \text{and also for}\ y \to \infty, \tag{6}$$

$$T = T_w(x), \ \text{at}\ y = 0, \quad T \to T_\infty, \ y \to \infty, \tag{7}$$

where the free-stream velocity U_∞, the stretching velocity U_w and the surface temperature T_w, are defined as

$$U_\infty = ae^{x/L}, \quad U_w = be^{x/L}, \quad T_w = T_\infty + ce^{x/L}, \tag{8}$$

where a, b and c are appropriate dimensional constants, and L is the reference length.

The significant physical measures of heat flux at the surface of the sheet q_w and the local Nusselt numbers Nu related with the current problem are

$$q_w = -k\frac{\partial T}{\partial y}|_{y=0}, \quad Nu_x = -\sqrt{\mathrm{Re}_x}\,\theta'(0), \tag{9}$$

where $\mathrm{Re}_x = U_\infty x^2 / 2\nu L$.

3. Solution of the Problem

For homotopy analysis solution of the flow problem take the following similarity transformations [16].

$$u = ae^{x/L} f'(\eta), \tag{10}$$

$$v = -(\frac{\nu a}{2L})^{1/2} e^{x/L} (f(\eta) + \eta f'(\eta)), \tag{11}$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \eta = \left(\frac{a}{2\nu L}\right)^{1/2} e^{x/2L} y. \tag{12}$$

With the help of transformations (10) to (12), $Eq.(1)$ is identically satisfied while $Eqs.(2-3)$ take the form

$$\lambda f^{(5)} - f''' - ff'' + 2f'^2 - 2 = 0, \tag{13}$$

$$\theta'' + \Pr(f\theta' - 2f'\theta) = 0, \tag{14}$$

where $\lambda = \eta_0 U_\infty / 2\mu\nu L$ is the nondimensional couple stress parameter. The boundary conditions in nondimensional form are stated as

$$f(0) = 0, \quad f'(0) = \varepsilon, \quad f''(0) = 0, \tag{15}$$

$$f' \to 1, \quad f'' \to 0, \ \text{as}\ \eta \to \infty, \tag{16}$$

$$\theta(0) = 1, \quad \theta \to 0, \ \text{as}\ \eta \to \infty, \tag{17}$$

where $\varepsilon = b/a$. The homotopy analysis method is highly dependent about the choice of the initial guesses and the corresponding auxiliary linear operators taken as [32-35]

$$f_0 = (\varepsilon - 1) + (\varepsilon - 1)\left(\frac{\eta^2}{2} - 1\right)e^{-\eta} + \eta, \tag{18}$$

$$\theta_0 = e^{-\eta}, \tag{19}$$

$$L_f = \frac{d^5}{d\eta^5} + 3\frac{d^4}{d\eta^4} + 3\frac{d^3}{d\eta^3} + \frac{d^2}{d\eta^2}, \tag{20}$$

$$L_\theta = \frac{d^2}{d\eta^2} + \frac{d}{d\eta}, \tag{21}$$

The results obtained are discussed in the next section.

4. Results and Discussion

The problem of the boundary layer stagnation flow and heat transfer of the couple stress fluid over an exponentially stretching surface is solved using the useful analytical technique homotopy analysis method (HAM). The HAM solutions are critically dependent upon the choice of auxiliary parameters $\hbar's$. Figures 1-3 are framed to examine the convergence regions for the auxiliary parameters $\hbar_1$ and $\hbar_2$ for velocity and temperature profiles for specified choices of the involved parameters. Figure 1 is sketched to observe the convergence region for velocity profile f' for $\eta = 0$, when couple stress fluid parameter $\lambda = 2$ and for different choices of the stretching parameter ε, plotted at the 15^{th} order of approximation of the HAM solution. From Figure 1 it is observed that the convergence region decreases with the increasing deviation of ε from 1.

Table 1. *Pade table showing the convergence of the velocity and temperature profiles for $\hbar_1 = \hbar_2 = -1$ when $\lambda = 1$, for $\theta'(0), \varepsilon = 1$.*

Homotopy-Pade approximation	f´´(0)		Homotopy-Pade approximation	Θ´(0)	
	ε = 0.75	ε = 1.25		Pr = 0.7	Pr = 2
[1/1]	0.25266	-0.26702	[1/1]	-1.33720	-1.74074
[2/2]	0.24664	-0.25719	[2/2]	-1.33518	-2.01060
[5/5]	0.23657	-0.24931	[7/7]	-1.33516	-2.25064
[6/6]	0.23626	-0.24902	[8/8]	-1.33516	-2.25384
[9/9]	0.23600	-0.24875	[12/12]	-1.33514	-2.25661
[10/10]	0.23597	-0.24873	[13/13]	-1.33514	-2.25672
[12/12]	0.23595	-0.24870	[16/16]	-1.33513	-2.25676
[15/15]	0.23595	-0.24870	[20/20]	-1.33513	-2.25676

The convergence region for shrinking sheet case with $\varepsilon = -0.25$ is $-0.8 \le \hbar_1 \le -0.2$. Figures 2-3 are prepared to observe the convergence regions for temperature profile θ. Figure 2 shows the convergence regions for different values of stretching sheet parameter ε and Prandtl number $\Pr$ when the couple stress parameter $\lambda = 1$, while Figure 3 contains the $\hbar$- curves for different values of the Prandtl numbers $\Pr$ and λ when $\varepsilon = 0$. From these plots it is observed that with increase in both $\Pr$ and λ the convergence region decreases [36-41].

Table 2. *Behavior of boundary derivatives for velocity profile.*

ε\λ	$f''(0)$					
	0.00	0.25	0.50	1.00	1.50	2.00
-0.50	4.98439	3.75931	2.95504	2.07307	1.66094	1.43591
-0.25	4.56249	3.47403	2.75245	1.94818	1.56296	1.34869
0.00	3.95760	3.03530	2.4190	1.72328	1.38388	1.19261
0.25	3.18567	2.45701	1.96695	1.4081	1.13161	0.974332
0.50	2.26074	1.75142	1.40703	1.01109	0.813047	0.699602
0.75	1.19521	0.929273	0.74865	0.539593	0.434127	0.373393
1.25	-1.08664	-0.91273	-0.77913	-0.59718	-0.48755	-0.41871
1.50	-2.26227	-1.90365	-1.62754	-1.2502	-1.02172	-0.87759
2.00	-4.2596	-3.73144	-3.29845	-2.65225	-2.21525	-1.91517
3.00	-7.97091	-7.28164	-6.67904	-5.69204	-4.93754	-4.35899

The influence of different involved parameters over velocity and temperature profiles is sketched in Figures 4-7. Figure 4 predicts the influence of couple stress parameter λ for velocity profile f' when $\varepsilon = 0.5, 1.5$. From Figure 4 it is observed that near the surface of the sheet, velocity profile decreases with increase in λ, while velocity profile changes its behavior and becomes increasing with increase in λ in far field from the surface when $\varepsilon = 0.5$, whereas the observed behavior for the velocity profile for $\varepsilon > 1$ is opposite to the case of $\varepsilon < 1$, that is velocity profile near the surface of the sheet is increasing with increase in λ, while in far field the observed behavior is decreasing. The turning point of the behavior for $\varepsilon = 0.5$ is nearly at $\eta = 2.5$, while for $\varepsilon = 1.5$ the point is somewhere at $\eta = 2$. Figure 5 is framed to check the accuracy of the HAM solutions for the second derivative boundary conditions. The curves in Figure 5 are sketched for different values of the stretching parameter ε and couple stress parameter λ. From Figure 5 it is observed that the pattern adopted is similar to that of velocity profile with a difference that the turning points in these curves are achieved much before then that for the curves in Figure 4 Figure 6 is included to check the influence of Prandtl numbers $\Pr$ and stretching ratio ε when the couple stress parameter $\lambda = 1$. Figure 6 inculcates that with increase in both ε and $\Pr$ the temperature profile and the thermal boundary layer thickness decreases. Figure 7 is graphed to examine the influence of λ over temperature profile θ. From this figure, it is given that with increase in λ the temperature profile and the thermal boundary layer thickness increases. Figure 8 gives the behavior of local Nusselt numbers for couple stress fluid calculated for different values of stretching parameter ε and local Reynolds numbers Re against Prandtl number Pr at the surface of the sheet. It is clear that with increase in each of ε, Pr and Re_x, local Nusselt numbers Nu increases.

To ensure convergence of the HAM solution, pade approximations for velocity and temperature profiles are also computed. Table 1 covers the pade approximation values for different values of ε, λ and Pr for velocity and temperature profiles computed up to $[25/25]$ iterations. The pattern followed by pade iterates guarantees the convergence of the HAM solution. It is also worthy to note that the convergence rate for velocity profile is much faster than that for temperature gradient. Table 2 provides behavior of boundary derivatives for velocity profile against different sets of ε and λ. The variation of heat flux at the surface of the sheet for a couple stress fluid is given in Table 3 for different pairs of ε and Pr. From Table 3 it is noted that increase in both ε and Pr produces a corresponding increase in the heat flux at the surface.

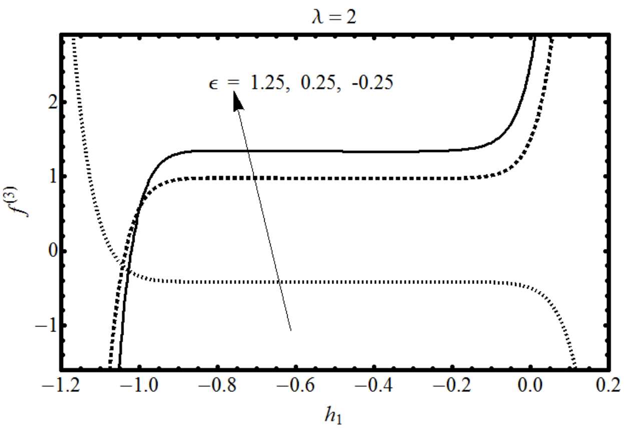

Figure 1. *ℏ - Curves for f' for different values of ε plotted at 15^{th} order of approximation.*

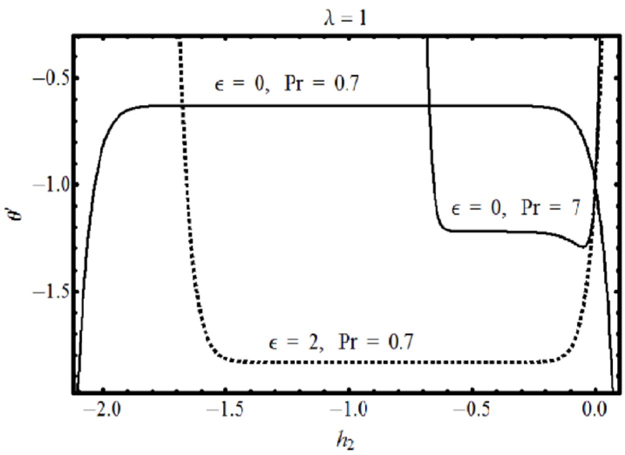

Figure 2. *ℏ - Curves for temperature profile θ for different values of ε and Pr, plotted at 20^{th} - Order of approximation.*

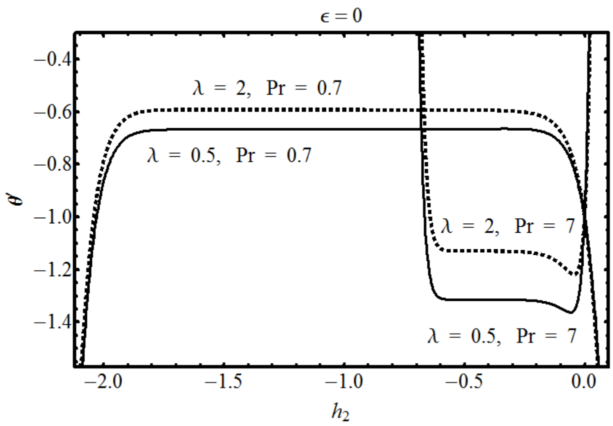

Figure 3. *ℏ - Curves for temperature profile θ for different values of λ and Pr, plotted at 20^{th} - Order of approximation.*

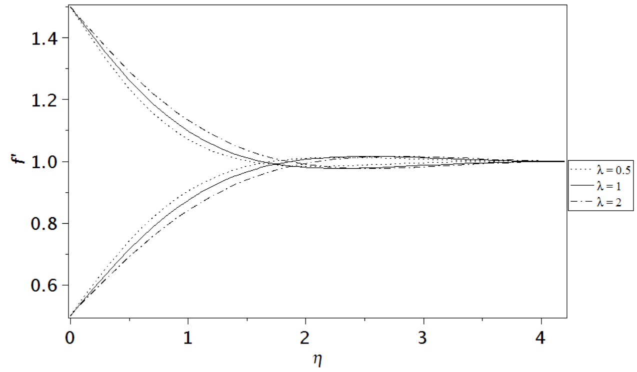

Figure 4. *Influence of λ over f' for $\varepsilon = 0.5, 1.5$.*

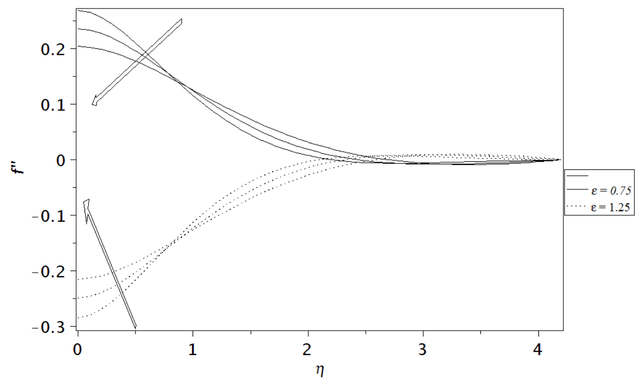

Figure 5. *Influence of ε over f'' for $\lambda = 0.5, 1, 2$.*

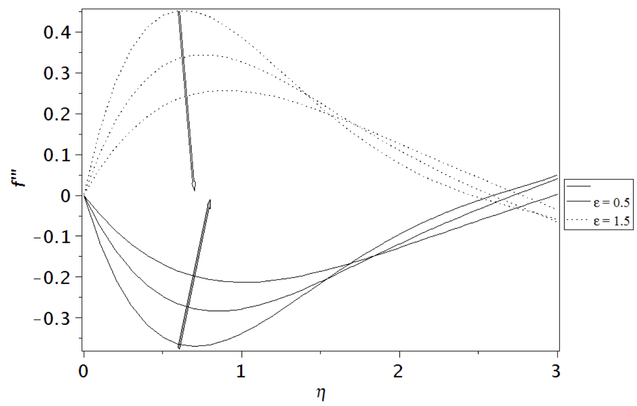

Figure 6. *Influence of ε over f''' for $\lambda = 0.5, 1, 2$.*

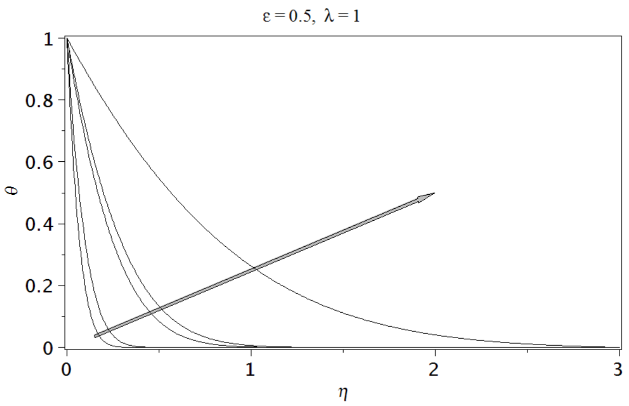

Figure 7. *Influence of* Pr *over* θ *for* $\mathrm{Pr} = 150, 70, 10, 7, 0.7$.

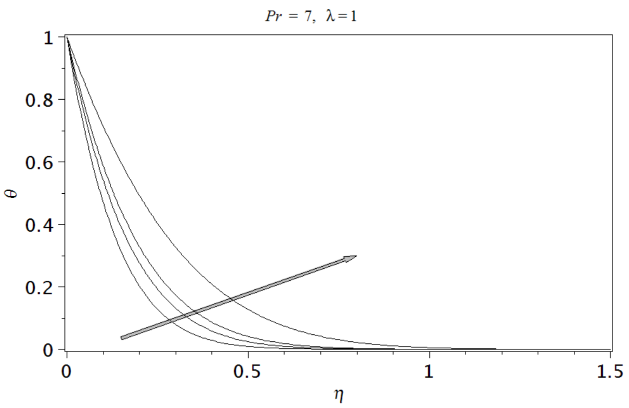

Figure 8. *Influence of* ε *over* θ *for* $\varepsilon = 3, 2, 1.5, 0.5$.

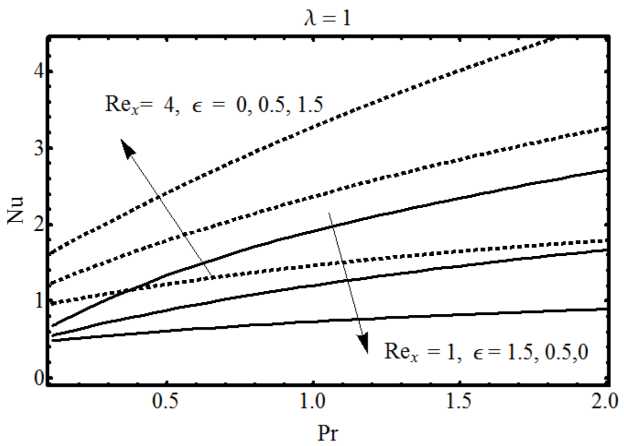

Figure 9. *Behavior of local Nusselt numbers for different involved parameters.*

Table 3. *Behavior of heat flux at the surface of the sheet when* $\lambda = 1$.

Pr\ε	-Θ′(0)				
	-0.25	**0.00**	**0.50**	**1.50**	**2.00**
0.2	0.45222	0.51496	0.63405	0.84997	0.94845
0.72	0.47004	0.66715	1.03252	1.64298	1.85047
1	0.49145	0.73116	1.20251	1.91382	2.19624
2	0.51054	0.89407	1.66341	2.71309	3.11933
5	0.52976	1.14246	2.55754	4.30189	4.95027
7	0.57571	1.24986	3.00875	4.93391	5.66433
10	0.60164	1.37531	3.56052	5.83020	6.50605
15	0.67469	1.42095	5.98810	6.25155	7.19630

5. Conclusion

Main findings obtained from the analysis are

1) By increasing the couple stress parameter λ the velocity profile f' decreases when $\varepsilon = 0.5, 1.5$.
2) By increasing the couple stress parameter λ the temperature profile θ increases.
3) By increasing both ε and Pr the corresponding heat flux at the surface increases.

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